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**SINGLE PHOTON HOLOGRAPHIC QUDIT ELEMENTS FOR LINEAR  
OPTICAL QUANTUM COMPUTING**

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FLORIDA ATLANTIC UNIVERSITY

*MAY 2011*

FINAL TECHNICAL REPORT

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## 1 SUMMARY

The operational objective of this project is to demonstrate that a small, lightweight, field-deployable, low-cost quantum algorithm (e.g. Grover's search algorithm) is feasible using existing technology. We have designed such optical elements and have begun a collaboration with Leonid Glebov of the The College of Optics & Photonics (CREOL) to manufacture such gates for experimental demonstration of the results under this effort. Our technical objective of this project was the simulation and design of fundamental quantum logic gates, each encoded as a multiplexed volume hologram (VH) within a single piece of Photo-Thermal Refractive (PTR) glass. The approach we used to accomplish this is (1) the simulation of a volume hologram on Air Force Research Laboratory (AFRL) General Purpose Graphical Processing Unit (GP/GPU) machines using a split-operator method, (2) mathematical modeling based on extensions of the coupled-mode theory, as well as (3) verification and visualization (V&V).

## 2 INTRODUCTION

Various approaches have been proposed for quantum computation. The most familiar of these is the quantum circuit model (QCM) [1]. Here the challenge is to (1) simulate those families of unitary operations on  $n$  quantum bits (qubits) that scale polynomially in  $n$  by small quantum circuits, and (2) project the readout onto the computational basis and measure the "output state." One application of this quantum circuit model involves linear optical quantum computing (LOQC) [2] and one-way quantum computing (OWQC) [3]. These are also known as cluster state quantum computing (CSQC) [4]. The essential feature of these approaches involve either a non-linear measuring process and/or the preparation of a highly-entangled  $n$ -qubit initial computational state. Here a sequence of measurements are made, projected onto one or another of the mutually-unbiased bases (MUBs). In our research we examined the utility of higher dimensional state spaces, i.e quantum d-level states (qudits), for LOQC. In addition to examining the potential strengths and weaknesses of qudit LOQC we extend our previous developments [5] in optical volume holography and designed and simulated practical single-photon, single-optical elements for qudit MUB-state quantum information processing. This includes the design of accurate MUB sorters and other relevant qudit unitary operators for LOQC. Our designs are based on coupled-mode analysis and its extensions; however, this approximation was further verified by numerical modeling. We developed a split-operator method running on AFRL GP/GPU computers.

An increase in the dimension of state space for quantum information processing (QIP) can increase tolerance to bit error rate (BER) while also increasing its bandwidth. The potential of extending photon-based QIP to higher dimensions was made popular in 1992 when Allen et al. [8] showed that Laguerre-Gaussian light beams possessed a quantized orbital angular momentum (OAM) of  $\hbar$  per photon. In 1983 Miller and Wheeler were the first to introduce fundamental quantum experiments using photons in OAM quantum states [9]. This opened up an arbitrarily high dimensional quantum space to a single photon [10]. Following these discoveries Mair et al. [11,12] unequivocally demonstrated the quantum nature of photon OAM by showing that pairs of OAM photons can be entangled using parametric down conversion. Shortly thereafter, Molina-Terriza et al. [13] introduced a scheme to prepare photons in multidimensional vector states of OAM commencing OAM QIP.

While photons with specific values of OAM have been emphasized in the literature we can equally well utilize any other set of orthogonal basis functions for higher-dimensional quantum

key distribution (QKD). While OAM states respect azimuthal symmetry, linear momentum (LM) states respect rectilinear symmetry. Independent of the representation we use, the MUB states will ordinarily be modulated in both amplitude and phase. Recently a practical method has been demonstrated to produce such MUB states using computer-generated holography with a single spatial light modulator (SLM) [6].

A significant obstacle for quantum computing with qudits ( $d \geq 3$ ) has been an efficient and practical quantum state sorter for photons whose complex fields are modulated in both amplitude and phase. We propose here such a sorter based on a multiplexed thick hologram, constructed e.g. from PTR glass, and demonstrate how such elements can be used to construct a practical quantum gate. We will validate this approach using coupled-mode theory with parameters consistent with PTR glass and simulate and validate the holographic sorter numerically. Our models will assume both two and three-dimensional state space spanned by three tilted plane waves. We extend our analysis to three dimensions, as this is one more dimension than available with photon polarization states. We will explore the utility of such a sorter for broader QIP and we believe that its impact can be substantial to quantum computing.

### **3 METHODS, ASSUMPTIONS AND PROCEDURES**

#### **3.1 Four Metrics and Milestones and Accomplishments.**

We addressed four primary metrics throughout this research effort. First, (Metric 1) we developed a split-operator code to provide an explicit simulation (split-operator method) and analytic model (coupled-mode theory) of a fundamental quantum logic gate recorded in a single piece of PTR glass. In particular, we designed and solved via coupled-mode theory the function of a volume-holographic Pauli-z gate as well as the quantum NOT gate and a quantum-state sorter. These coupled-mode results were further verified with a split operator method as well as a finite-difference time-domain (FDTD) code from Massachusetts Institute of Technology's (MIT's) MIT electromagnetic equation propagation code (Meep). Secondly, (Metric 2) all simulations were suitably benchmarked against analytic and known solutions and our simulations were run on AFRL high performance computing (HPC) GP/GPU and message passing interface (MPI) platforms. Our undergraduate student (UG) student (Chris Tison) worked under the summer intern program at AFRL's Emerging Computing Technology Group (RITC) and migrated our codes to the AFRL/RITC Nvidia GPU cluster. We collaborated with Leonid Glebov's group at the University of Central Florida. Together we ascertained that multiplexing with high fidelity was indeed possible in photo-thermal refractive (PTR) glass. Furthermore, the CREOL research group determined that it takes roughly one millimeter of PTR glass per exposure in order to achieve high fidelity multiplexed gratings. Furthermore, PTR holograms can approach thicknesses of 2 cm. Therefore, we can, in principle, compute in a 20-dimensional quantum state space. This limits this technology to a linear optical quantum computer such that each quantum gate has to use less than five or fewer qubits. It is well known that this technology is not scalable; however, there may be many quantum information processing tasks that may require a relatively small ( $\leq 5$ ) amount of bits (i.e. quantum error correction). These findings, together with our designs, models, and simulations, demonstrate that a small, lightweight, field-deployable, low-cost quantum algorithm (e.g. Grover's search algorithm) is feasible using existing technology. Finally (Metric 4), we have a paper accepted for the March 2011 International Society for Optics and Photonics (SPIE) Defense, Security and Sensing meeting in Quantum Information and Computation IX. (Paper number

8057-27, 29 Aug 2011.). In this paper the simulation of a fundamental quantum logic gate will be submitted for public release and publication in a refereed journal.

We met or exceeded all four metrics and milestones described above. In particular, we designed and simulated a high-fidelity volume hologram (VH) qubit logic gate. Our simulations were done both at Florida Atlantic University (FAU), and at AFRL during the summer by an FAU UG student (Chris Tison). We leveraged this AFRL grant and received approval from FAU to purchase a 7 teraflop GPU computer for student training purposes. This Nvidia computer is similar to the Nvidia machines used at AFRL. This enabled our student to immediately show progress under the summer internship at AFRL. In order to continue this research the FAU Principal Investigator (PI) and United States (US) student were given H-token access to AFRL compute facilities. Both the PI and student (C. Tison) spent the summer at AFRL's Information Directorate (RI). During this grant period we successfully provided a GPU simulation of a VH quantum gate using our paraxial split operator code. As a follow-on to this grant, we presented a specific design for the fabrication of a PTR qubit VH at a December 2010 kickoff meeting at CREOL with Leonid Glebov's group.

During this effort we analyzed two independent sets of quantum states of a photon that would allow us to explore higher-dimensional state spaces. One quantized the transverse LM (tilt of the wavefront) of the photon while the other quantized the photon's OAM. We extensively analyzed the efficiency of OAM verses LM volume holographic gratings and concluded that higher efficiencies could be reasonably achieved with LM states thus arguing against the use of photons with OAM.

### **3.2 Publications Under this Effort**

There are three publications in refereed journals associated to this effort, one poster contribution, and three seminars. The publications are as follows:

1. W. Miller, "Efficient Photon Sorter in a High Dimensional State Space," Journal Quantum Information and Computation, Vol. 11, No. 3&4 (2011) 0313.
2. SPIE Defense, Security and Sensing paper accepted. P. Alsing, G. Kreymerman, J. McDonald, W. Miller, and C. Tison, "Quantum Computing in a Piece of Glass," in Quantum Information and Computation IX. Paper number 8057-27, 29 Aug 2011.
3. P. Alsing, G. Kreymerman, W. Miller and C. Tison, Volume Holographic Gratings for Photons with Orbital Angular Momentum, in preparation.

## **4 RESULTS AND DISCUSSION**

### **4.1 Design of Optical Quantum Computing CNOT Gate**

Here we provide a specific design of a quantum controlled not (CNOT) gate in PTR glass. We could have chosen any 2 or 4-qubit gates, but chose this one to illustrate our design as it is a canonical example found in the literature. The CNOT gate is a 2-qubit gate. This is a  $2 \times 2$ , or 4-dimensional state space. This state space can be constructed as a product space of qubits, e.g. by utilizing the polarization states of two correlated photons. However, in a higher-dimensional state space such as that afforded by the LM photon states the CNOT gate can be constructed with a single photon. In our design, we choose four independent plane waves lying on the cone shown in Figure 1. We associate these independent transverse LM modes with the orthogonal quantum



state vectors,  $|a\rangle$ ,  $|b\rangle$ ,  $|c\rangle$  and  $|d\rangle$ , so that any state vector,  $|\psi\rangle$  in the 4-dimensional state space can be written as a linear superposition of these states,

$$|\psi\rangle = \alpha|a\rangle + \beta|b\rangle + \gamma|c\rangle + \delta|d\rangle, \quad (1)$$

with,

$$|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1. \quad (2)$$

Each of our basis states can be expressed in matrix notation,

$$|a\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, |b\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, |c\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \text{ and } |d\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}. \quad (3)$$

Then the CNOT gate can be expressed by the following unitary transformation on these basis vectors:

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}. \quad (4)$$

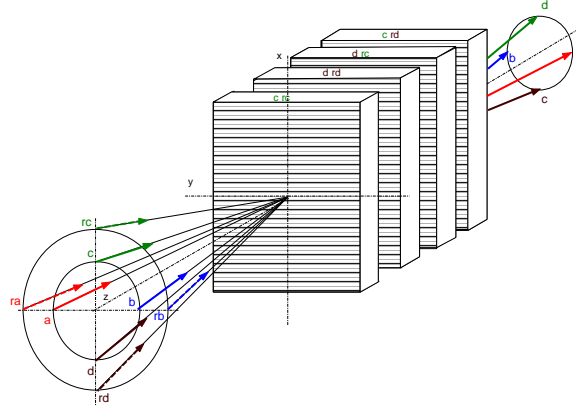


Figure 1: Volume holographic design of the 4-dimensional CNOT gate in PTR glass. The gate can be constructed by a stack of 4 LM gratings, or by a stack of two multiplexed gratings.

We let the  $z$ -axis be orthogonal to the face ( $x$ - $y$  plane) of the hologram. The four volume holographic gratings are recorded by a suitable superposition of the set of four signal plane waves,

$$\langle r|a\rangle = \exp(ik_a \cdot r), \langle r|b\rangle = \exp(ik_b \cdot r), \langle r|c\rangle = \exp(ik_c \cdot r), \text{ and } \langle r|d\rangle = \exp(ik_d \cdot r), \quad (5)$$

and four reference waves,

$$\langle r|r_a\rangle = \exp(ik_r a \cdot r), \langle r|r_b\rangle = \exp(ik_r b \cdot r), \langle r|r_c\rangle = \exp(ik_r c \cdot r), \text{ and } \langle r|r_d\rangle = \exp(ik_r d \cdot r), \quad (6)$$

as shown in Fig. 1.

The hologram is recorded so that each row of the unitary matrix of the CNOT gate (or any unitary gate of our choice) is used to generate its own volume holographic grating. For a 2-qubit gate such as the CNOT gate we would ordinarily require four recordings; however, since the first two bits involve just an identity matrix we need only two layers to transform the signal states into the desired reference states. In addition to one holographic recording per dimension of the state space, we also require the conjugate of each grating (two in the case of the CNOT gate) in order to transform the diffracted reference waves back into the desired signal states. In particular, the CNOT-gate is constructed from four holographic gratings stacked together as illustrated in Fig. 1. In particular, this can be achieved by the following four stacked gratings:

1. the first grating is recorded with the two coherent plane waves corresponding to states  $|c\rangle$  and  $|r_c\rangle$  ;
2. the second grating is recorded with the two coherent plane waves corresponding to states  $|d\rangle$  and  $|r_d\rangle$  ;
3. the third grating is recorded with the two coherent plane waves corresponding to states  $|r_d\rangle$  and  $|d\rangle$  ; and
4. the fourth grating is recorded with the two coherent plane waves corresponding to states  $|r_c\rangle$  and  $|c\rangle$  .

None of the four gates diffract states  $|a\rangle$  or  $|b\rangle$ . The first two gratings redirect the two signal states  $|c\rangle$  and  $|d\rangle$  into  $|r_c\rangle$  and  $|r_d\rangle$  according to the Pauli  $x$ -gate,

$$\hat{X} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad (7)$$

and is equivalent to the operator,

$$\hat{R}_1 = |a\rangle\langle a| + |b\rangle\langle b| + |r_c\rangle\langle c| + |r_d\rangle\langle d|. \quad (8)$$

The second two holograms are equivalent to the operator,

$$\hat{R}_2 = |a\rangle\langle a| + |b\rangle\langle b| + |d\rangle\langle r_d| + |c\rangle\langle r_c|. \quad (9)$$

Therefore, the combination of the four stacked volume holograms have the desired action – the CNOT gate.

$$CNOT = \hat{R}_2 \cdot \hat{R}_1 = |a\rangle\langle a| + |b\rangle\langle b| + |d\rangle\langle c| + |c\rangle\langle d|. \quad (10)$$

We have applied these principles to design a universal set of quantum gates.

## 4.2 The efficiency of an OAM and LM Volume Grating

We modeled the diffraction efficiency of a volume hologram constructed by the coherent recording of (1) a Laguerre-Gaussian OAM beam aligned on axis, with (2) a plane reference wave as shown in Fig. 2. Then let  $\theta$  be the angle between the normally-incident OAM beam and the tilted plane reference wave. In particular, the hologram is recorded with the interference with reference wave  $R$  and Laguerre-Gaussian signal wave  $S_+$ . In Fig. 3 we provide a color contour plot of the index of refraction recorded into the emulsion at a depth of  $z = 500\lambda$ . Again, this volume hologram was

produced by the interference of (1) a normally-incident Laguarre-Gaussian signal state with unit OAM, and (2) a plane wave tilted at an angle of  $\theta = \pi/14$  with respect to the normal. This is well within the paraxial approximation. The waist of the  $LG_0^1$  signal was  $50\lambda$  as is apparent from the diameter of the contour plot of the index of refraction shown in Fig. 3.

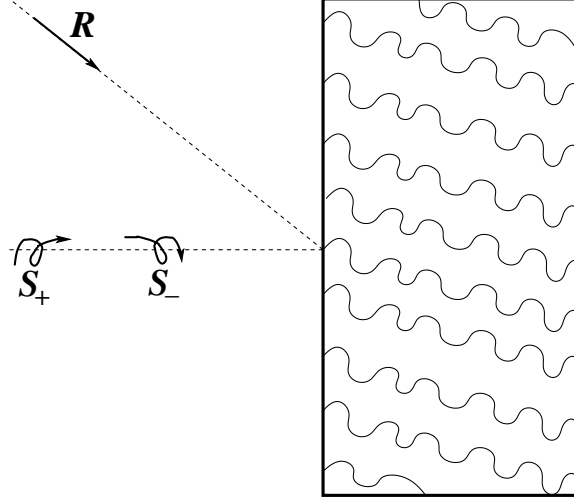


Figure 2: OAM volume hologram sorter.

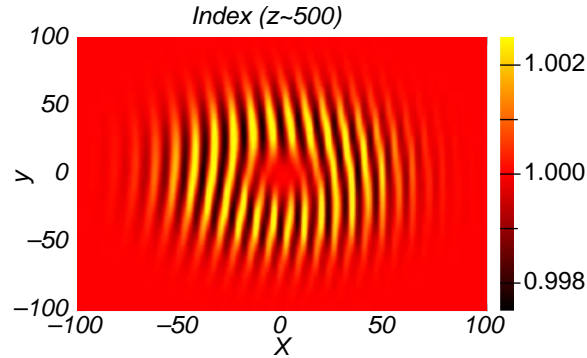


Figure 3: Contour plot of the index of refraction.

With our split-operator code, and with our finite-difference time domain code we can analyze the electric field as it propagates through the volume hologram and is diffracted. This helps us compare our results with other published runs and to understand the physical mechanism of diffraction. In Fig. 4 we show the intensity of a Laguarre-Gaussian beam beam after propagating through a matched OAM grating, i.e. the recording state is identical to the reconstruction state. On the left we show the intensity for a pure Gaussian-beam grating ( $LG_0^0$ ) with vanishing OAM. On the right in Fig. 4 we show the intensity for a beam with  $\hbar$  of OAM,  $LG_0^1$ . Our paraxial code reproduced earlier results, and predicts substantial qualitative differences for OAM gratings [16].

To measure the sorting efficiency of such a volume holographic grating, we assume that an equal and random ensemble of signal states matched ( $S_+$ ) and mismatched ( $S_-$ ) is directed into the volume hologram. We then count the bit error rate (BER). Here we define the BER as the error

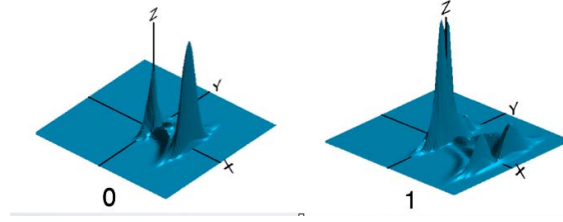


Figure 4: Intensity for (left) Gaussian-beam, (right)  $L = 1$  LG beam.

rate in diffraction on a photon-by-photon level. In particular, the BER is the rate at which (1) the mismatched states are diffracted incorrectly into  $R$ , plus (2) the rate at which the matched signal state, which ordinarily should be diffracted, is actually not diffracted. With this measure of BER, a completely random, i.e. useless sorter will yield a 50% BER, while a perfect sorter would give 0% BER. Fig. 5 shows the predicted BER of a volume-holographic OAM grating as a function of the thickness,  $z$ , of the holographic emulsion as measured in units of wavelengths. Here the grating is recorded using a pure Gaussian beam with no OAM. It is assumed to be reconstructed with an equal ensemble of (1) matched Gaussian signals, and (2) a mismatched OAM signal with  $L = \{1, 2, 4, 8\}h$ . Here we plot four separate curves plotting the rate of error sorting the OAM mismatch,  $\Delta L = 1$  (solid curve),  $\Delta L = 2$  (dotted curve),  $\Delta L = 4$  (dashed curve), and  $\Delta L = 8$  (dashed-dotted curve). The sorting improves, and the BER diminishes when the difference in OAM of the signal and recording wave gets larger. For  $\Delta L = 8$  we can achieve greater than 90% sorting efficiency.

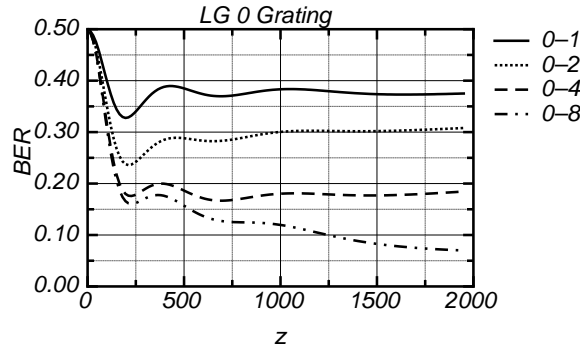


Figure 5: Predicted bit error rate (BER).

### 4.3 Coupled-Mode Analysis of a Single OAM Grating

It is sufficient here to thoroughly analyze a single optical vortex grating and to exhibit its ability to differentiate a photon with one value of OAM from another. Once this has been demonstrated, the higher-dimensional sorters can be constructed by literally stacking or sequencing such gratings, one for each basis state in a given MUB. This can also be accomplished by incoherently multiplexing each of the gratings into a single holographic substrate. We examined in this project a holographic emulsion produced by the coherent interference of a unit-amplitude plane wave and the unit amplitude OAM wave front corresponding to  $l = 1$ . The intensity of the interference pattern (Fig. 2),

$$I = \left| e^{i\vec{k}_r \cdot \vec{r}} + e^{i(\vec{k}_s \cdot \vec{r} + l\phi)} \right|^2, \quad (11)$$

will be recorded in the emulsion of bulk index  $n_0$  and depth of modulation  $\delta n \ll n_0$  to yield an index of refraction variation,

$$n = n_0 \left( 1 + \frac{\Delta n}{n_0} \cos \left( (\vec{k}_r - \vec{k}_s) \cdot \vec{r} - l\phi \right) \right). \quad (12)$$

We wish to find an approximate solution of the Helmholtz equation,

$$\nabla^2 E + k^2 E = 0. \quad (13)$$

We solved this equation using three independent approaches; (1) a FDTD algorithm, (2) the split-operator method to solve the paraxial wave equation, and (3) a semi-analytic approach using a modified coupled-mode theory. The results of each of these approaches, as applied to both OAM gratings as well as LM gratings were shown to be convergent and consistent (Figs. 4-6). Using coupled-mode theory on an orbital angular momentum (OAM) volume Bragg grating one can avoid large-scale numerical computations.

We consider here only two distinct modes of a  $y$ -polarized electric field  $E$ , one consisting of a plane reference wave with wave vector  $\vec{k}_r$  and one with orbital angular momentum (OAM) propagating in the  $z$ -direction with wave vector  $\vec{k}_s = k\hat{z}$  and angular momentum  $l$ .

$$E = R e^{i\vec{k}_r \cdot \vec{r}} + S e^{i(\vec{k}_s \cdot \vec{r} + l\phi)} \quad (14)$$

Here  $R(\vec{r})$ ,  $S(\vec{r})$  are the amplitudes of reference and signal modes, respectively.

As is customary with coupled-mode theory, we keep only these two modes in our calculation and neglect the second derivative terms. Consequently the first term of Eq. 13 contains two primary modes.

$$\nabla^2 E \approx \left( 2i\vec{k}_s \cdot \nabla S + \underbrace{2i\frac{\vec{l}}{\rho^2} \cdot (\vec{r} \wedge \nabla S) - \frac{l^2}{\rho^2} S - \beta^2 S}_{\text{vanishes for large } \rho} \right) e^{i(\vec{k}_s \cdot \vec{r} + l\phi)} + \quad (15)$$

$$\left( 2i\vec{k}_r \cdot \nabla R - \beta^2 R \right) e^{i\vec{k}_r \cdot \vec{r}} + \text{other modes} \quad (16)$$

Here we ignore the second-order derivatives and we defined an OAM vector,

$$\vec{l} = l\hat{k}_s. \quad (17)$$

We now examine the second term of the Helmholtz equation (Eq. 13) within the approximations of coupled-mode theory.

$$k^2 E \approx n^2 \left( \frac{\omega}{c} \right)^2 E = n^2 k_0^2 E. \quad (18)$$

This expression can be approximated by neglecting the higher-order terms and keeping only our primary two modes of Eq. 14.

$$k^2 E \approx \beta^2 E + \beta^2 \frac{\Delta n}{n_0} \left( S e^{i\vec{k}_r \cdot \vec{r}} + R e^{i(\vec{k}_s \cdot \vec{r} + l\phi)} + \text{other modes} \right) + O \left( \frac{\Delta n^2}{n_0^2} \right) \quad (19)$$

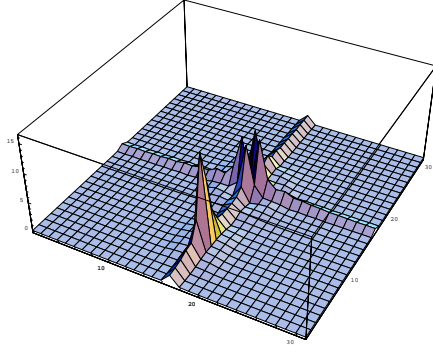


Figure 6: Farfield diffraction pattern of the solution of Eqns. 20 & 21.

Substituting the approximate expressions for  $\nabla^2 E$  and  $k^2 E$  into Eq. 13, imposing the Bragg matching condition ( $\beta = |\vec{k}_s| = k = n_0 k_0$ ) and setting the coefficients of each of the two primary modes equal to zero we obtain the two coupled-mode partial differential equations for the amplitudes  $R$  and  $S$ .

$$i\vec{k}_s \cdot \nabla S + \underbrace{2i\frac{\vec{l}}{\rho^2} \cdot (\vec{r} \wedge \nabla S) - \frac{l^2}{\rho^2} S + \beta^2 \left( \frac{\Delta n}{n_0} \right)}_{\text{vanishes for large } \rho} R = 0, \quad (20)$$

and

$$2i\vec{k}_r \cdot \nabla R + \beta^2 \left( \frac{\Delta n}{n_0} \right) S = 0. \quad (21)$$

We immediately see that for large values of the cylindrical coordinate  $\rho \gg \lambda$  these equations reduce to the usual thick Bragg grating coupled-mode equations as they should.

Again, in the case of the OAM grating, we solve numerically Eqs. 20-21 and compare these solutions with the results of a fully 3-dimensional finite-difference time-domain solution of Maxwell's equations. For example, in Fig. 7 we display the results of a 3-dimensional simulation of an OAM grating using the finite-difference time-domain code Meep. The grating we simulated is produced by the interference of a normally-incident  $l = 1$  optical vortex signal ( $S$ ) and a reference plane wave ( $R$ ) tilted at 30 deg. The bulk index of refraction,  $n = 1$ , is modulated with amplitude,  $\Delta n = 0.02$ . The hologram we simulated had transverse dimensions of  $40 \times 40$  in units of wavelength and was 30 wavelengths in thickness. We used a grid resolution of 20 grid points per wavelength, which represents a computation grid with  $800 \times 800 \times 600$  grid points. The solid (red) curve represents the fraction of the incident wave diffracted at 30 deg in the direction of the reference beam, while the dashed curve represents the DC, or undiffracted, component. At approximately 25 wavelengths into the emulsion the efficiency of the grating is greater than 92%. This was the maximum resolution our cluster could simulate. However, based on similar 2-dimensional calculations of Bragg gratings we would require higher resolutions in order for our numerical simulations to converge. Nevertheless, we expect slightly (a few percent) higher efficiencies than we reported here.

Throughout this research we examined gratings with a broad range of depth of modulation

in the index of refraction ( $\Delta n \in \{0.1, \dots, 0.0005\}$ ), and since Maxwell's equations are scale-invariant, we presented our solutions scaled in units of wavelength,  $\lambda = 1$ .

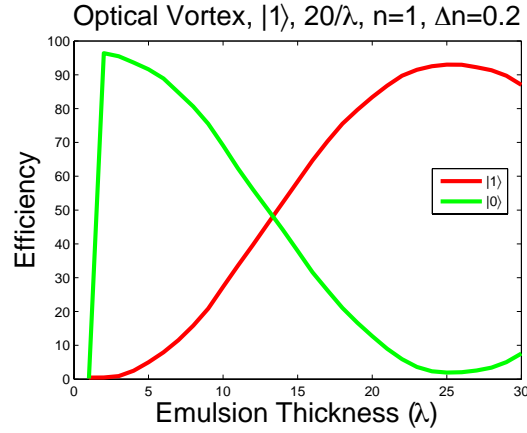


Figure 7: A 3D FTDT simulation of the OAM grating.

#### 4.4 Coupled-Mode Analysis of the LM Sorter

For the detailed analysis of the coupled-mode analysis of the LM sorter, which is at the foundation of the construction of the volume holographic quantum gates, the reader is referred to the recent publication under this effort [5].

## 5 CONCLUSIONS

The work presented here proposes a lightweight, environmentally robust device to elicit any desired quantum interference up to five qubits. It gives us a natural path for transitioning this research beyond this one-year exploratory project to the following milestones:

1. Design and simulation of a VH quantum algorithm that can replace a complex multiple optical element system with a single piece of PTR glass.
2. Explore the limits of multiplexing, i.e. answer the question, What is the highest dimensional state space we can feasibly explore with existing technology?
3. Fabrication and Demonstration (AFRL, FAU, Colgate and CREOL) of a PTR VH-based quantum algorithm.
4. Compare advantages of LM states verses OAM states.

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## 6 LIST OF SYMBOLS, ABBREVIATIONS AND ACRONYMS

AFRL	Air Force Research Laboratory
BER	Bit Error Rate
CNOT	Controlled Not
CREOL	The College of Optics & Photonics
CSQC	Cluster State Quantum Computing
FAU	Florida Atlantic University
FDTD	Finite Difference Time Domain
GP/GPU	General Purpose Graphical Processing Units
HPC	High Performance Computing
LM	Linear Momentum
LOCQ	Linear Optical Quantum Computer
Meep	Massachusetts Institute of Technology's Electromagnetic Equation Propagation Code
MIT	Massachusetts Institute of Technology
MPI	Message Passing Interface
MUB	Mutually Unbiased Basis
OAM	Orbital Angular Momentum
OWQC	One-Way Quantum Computer
PI	Principle Investigator
PTR	Photo-Thermal Refractive
QIP	Quantum Information Processing
QCM	Quantum Circuit Model
QKD	Quantum Key Distribution
qubit	Quantum bit
qudit	Quantum $d$ -level system
RI	Information Directorate
RITC	Emerging Computing Technology
SLM	Spatial Light Modulator
SPIE	International Society for Optics and Photonics
UG	Undergraduate
US	United States
VH	Volume Hologram
V&V	Verification and Visualization



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